Viola, **8th grade**
How long would it take an object weighing 613 kg to free fall through Mars' atmosphere before hitting the surface? How could I work out this problem taking place on Mars?

I will start off with some assumptions. 1) Since you say “atmosphere,” I will find the general solution for an arbitrary height, so that the starting point might be one where gravity is weaker but grows in strength as the object falls. 2) I will ignore air resistance, since the atmospheric pressure on Mars is less than 1% of Earth’s; this is an okay approximation, since the folks at NASA already have a lot of trouble making even parachutes that work with such little air. 3) I will assume that this mass starts stationary and then falls straight down. If there is any sideways motion, the equations complicate considerably, and if there’s enough sideways motion, the object might not hit Mars at all and will instead orbit like a satellite.

One important thing to note is that the one number you gave me actually does not enter the equations at all! You may know about Galileo’s famous experiment, when he dropped two objects of different mass from the Leaning Tower of Pisa and they landed at the same time. Mass only enters into the air resistance equation, which we ignore because it’s also a complicated function of the object’s shape (how aerodynamic it is).

Now that those considerations are out of the way, I derived this equation using the conservation of energy:

Starting potential energy = Final kinetic energy + Final potential energy

$$-\frac{GMm}{R+h} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

Here, *G* is the gravitational constant; *M* is the mass of Mars; *m* is the mass of the object; *R* is the radius of Mars (the final position when it hits the ground); *h* is the height off the surface at the beginning; and *v* is the final velocity. Since every term has *m* in it, I can divide it out, and we recover the fact that the mass of the object does not affect its trajectory, since the rest of the math from here on out does not depend on *m*. After a few steps involving algebra and calculus, we end up with this equation for *T* the time to hit the ground:

$$T = \frac{(R+h)^{3/2}}{\sqrt{2GM}}\left[\arcsin\left(\sqrt{\frac{h}{R+h}}\right) + \frac{\sqrt{Rh}}{R+h}\right]$$

This is related to Kepler’s equations for the motion of orbiting planets. The radius of Mars is 3.39 thousand kilometers, and its mass is 6.42×10^{23} kilograms. **If we start from the top of the atmosphere, at 250 km up, the fall would take a little under 6 and a half minutes.** It turns out you can get almost the same answer if you assume gravity is constant through the whole fall.